LARGE-SCALE STRUCTURES AND PERCOLATION THEORY

UMEMURA, M.

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M. UMEMURA

Tokyo Astronomical Observatory
University of Tokyo, Mitaka, Tokyo 181

ABSTRACT

The application of percolation theory to the analysis of large-scale structures in the Universe is proposed. It is found that the percolation analysis is an effective method in that it can discriminate not only the geometry but also the directionality of structures. This method is more powerful, if it is combined with Fractal analysis (two-point correlation function), nearest neighbor analysis, and pair separation analysis.

I. INTRODUCTION

Now, basically speaking, we have three kinds of mechanisms to form the large-scale structures in the Universe, that is, the so-called pancake theory (top-down scenario) (Zeldovich 1970), the hierarchical clustering theory (bottom-up scenario) (Peebles 1980), and the explosion theory (Ostriker and Cowie 1981; Ikeuchi 1981). In the pancake theory, at first a large-scale triaxial density fluctuation collapses toward the direction of the shortest axis, so that a pancake-like gaseous protocluster is formed. This pancake fragments into a number of galactic-scale lumps and a large-scale structure of galaxy distribution appears. In clustering theory, the structures form sequentially from the subgalactic-scale to the large-scale by the gravitational clustering. On the other hand, in the explosion theory, large-scale structures are formed by a non-gravitational mechanism, that is, the blast wave due to explosive energy release.

At present, we have not yet come to the final conclusion which scenario is the real history. It is in part because we have no decisive way to analyze the existing structures precisely and quantitatively and compare them with what is expected in each theory. The two-point correlation function has been so far one of effective probes (Totsuji and

Kihara 1969; Peebles 1980). In addition, the nearest neighbor distribution was used as a statistical quantity including not only two-point correlation but also the higher order correlation (White 1979). These methods, however, present just the averaged information concerning the geometry. The other aspects such as directionality, alignment, elongation, and so on, are smeared out. Bahcall *et al.* (1986) proposed another method, i.e., the pair separation distribution. This analysis extracts primarily the information concerning the systematic elongation in a certain direction.

Recently the Fractal theory and Percolation theory attract much attention in solid-state physics and geophysics, because they can analyze quantitatively the structures which seem very complicated at first sight. It is possible to apply these theories to the analysis of large-scale galaxy distributions. Here, I propose an application of such theories and compare the effectiveness with that of previously proposed analyses.

II. SEVERAL ANALYSES

a) Two-point Correlation and Fractal

The two-point correlation function has been one of major analyses of large-scale structures in the Universe. It is often expressed in terms of a power law as

$$\xi(r) \sim A r^{-\gamma}$$

(Totsuji and Kihara 1969; Peebles 1980; Davis and Peebles 1983; Bahcall and Soneira 1983). The power index γ is $\sim$ 1.8 for both galaxies and clusters of galaxies, while the amplitudes are significantly different, that is, $A_G \sim 20$ for galaxies and $A_C \sim 360$ for clusters of galaxies when r is in units of Mpc.

Such two-point correlation is related to the Fractal as follows. When a structure is Fractal, the mass or number within r is

$$M(r) = cr^D,$$

where D is called Fractal dimension, which is often non-integral. In a Fractal structure, the density at a radius r is

$$\rho(r) = \left(\frac{cD}{4\pi}\right) r^{D-3}.$$

On the other hand, the average density within a radius R of the sample is

$$\overline{\rho}(R) = \left(\frac{3c}{4\pi}\right) R^{D-3}.$$

Therefore the two-point correlation function is

$$\xi(r) = \frac{\rho}{\bar{\rho}} - 1 = \frac{D}{3}\left(\frac{r}{R}\right)^{D-3} - 1.$$

If $\xi \gg 1$, then

$$\xi(r) \simeq A r^{D-3},$$

where $A = DR^{3-D}/3$. This means that the power index γ of two-point correlation function is related with the Fractal dimension D as

$$\gamma = 3 - D.$$

The above-mentioned power $\gamma = 1.8$ for galaxies or clusters of galaxies corresponds to the Fractal dimension $D = 1.2$, which is intermediate between plane and filament.

The two-point correlation function turns out to be a good measure for the fundamental geometry of structure. Furthermore, Calzetti *et al.* (1987) pointed out that if our Universe is Fractal from galactic-scale to large-scale, then the observed difference between A_G and A_C can be accounted for in terms of the discrepancy of sample region size R by a factor ~ 5, because the amplitude is dependent upon R as seen the above equation.

b) Nearest Neighbor Distribution

The two-point correlation function includes no information on other correlations. The nearest neighbor distribution includes all orders of correlations. It is defined as

$$g(r) = \frac{1}{N_r} N(r_{nn} > r),$$

where $N(r_{nn} > r)$ is the number of test points whose distance to the nearest sample exceeds r (White 1979). This function $g(r)$ is the probability such that no sample is involved in a region within r:

$$g(r) = \exp\left[\sum_{j=1}^{\infty} \frac{(-n)^j}{j!} \int\!\!\int \cdots \int \xi_j(\mathbf{x}_1 \cdots \mathbf{x}_j)\, dV_1 \cdots dV_j\right],$$

where n is the mean number density and ξ_j is the j-point correlation function. Instead of $g(r)$, the function $1 - g(r)$ may be more convenient to see the structure intuitively. For example, a bar structure embedded in a low-dense region gives a power law $1 - g(r) \propto r^{\lambda}$ with $\lambda = 2$. An isolated sheet structure gives $\lambda = 1$.

The nearest neighbor distribution is another measure for the geometry of structure. Especially, this is an effective way to probe the high-order clustering and large vacant regions (White 1979).

c) Pair Separation Distribution

The above two methods are, fundamentally, measures for the geometry. They, however, don't inform us of the elongation and its direction of structures. Bahcall *et al.* (1986) proposed an attractive method, i.e., the pair separation distribution, which gives the information concerning the systematic elongation in a certain direction. They analyzed the projected separations of all sample pairs on α, δ directions and a line of sight, respectively, and found that large-scale structures are systematically elongated to a line of sight by a degree of $\sim$ 1400 km s^{-1} in cluster velocity dispersion.

d) Percolation

The concept of percolation has been used in a wide variety of solid-state or geophysical contexts (Stauffer 1985). For example, it is known that in an infinite medium the conductive percolation clusters become larger when the fraction of metal component is raised, and the conductivity increases drastically when the probability of blended metal exceeds a critical value, p_{crit}. This is interpreted that an infinite conductive percolation cluster is formed. The distribution of percolation clusters are known to be Fractal at such phase transition. In this sense, the percolation may be regarded a broader concept than the Fractal.

The percolation is applied to the analysis of large-scale galaxy distributions by Zeldovich *et al.* (1982) and Vettolani *et al.* (1986). Zeldovich *et al.* examined the maximum lengths of percolation clusters against the variable searching radius. Their analysis is beneficial for exploring the way of clustering. It, however, mainly focuses on the connectivity rather than the directionality of structures.

III. NEW PERCOLATION ANALYSIS

Recently, we (Umemura *et al.* 1987) had a new attempt of the application of the percolation theory. In our analysis, we define percolation clusters if the sphere of R_0 centered on each sample intersects another one, and calculate the projected lengths of percolation clusters onto each axis. Then the maximum values of projected lengths, DX, DY, and

DZ, and $DL = [(DX)^2 + (DY)^2 + (DZ)^2]^{1/2}$ are plotted against R_0. By this analysis, we can recognize the directionality of structures more precisely and quantitatively, because it is sensitive to the direction of structure.

The distances of galaxies or clusters of galaxies from us are estimated from the recession velocity assuming the ideal Hubble law. Therefore, if they have some velocity dispersion, the pseudo-deformation of distributions appears. The present percolation analysis gives also a measure for elongation in z-direction, that is

$$\beta \equiv \mathrm{Max}(\frac{2DZ}{DX + DY}). \tag{1}$$

If the value of β is specified, we can know the degree of elongation as corresponding velocity dispersion.

We applied this method to the observed spatial distribution of clusters of galaxies (Bahcall *et al.* 1986). The result for the data of $| l | < 90°$, $b > 30°$ is shown in Fig. 1, and that for the data of $| l | > 90°$, $b > 30°$ is shown in Fig. 2. It has been found that in the former region there is a large elongated structure of $\beta \sim 2.5$, which corresponds to the velocity dispersion of ~ 1000 km s^{-1}. In addition, there is an apparently geometrically elongated structure of ~ 150 Mpc in the latter region.

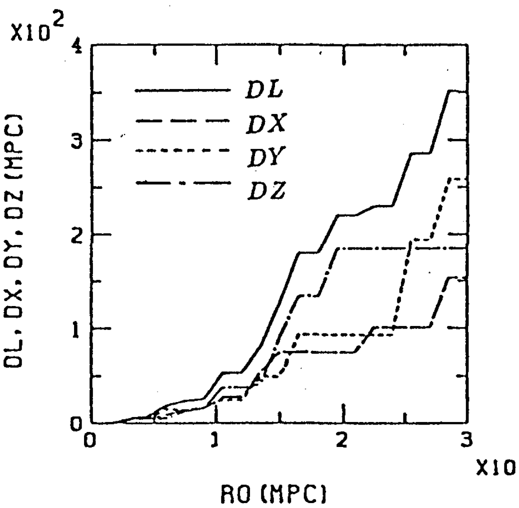

Fig.1

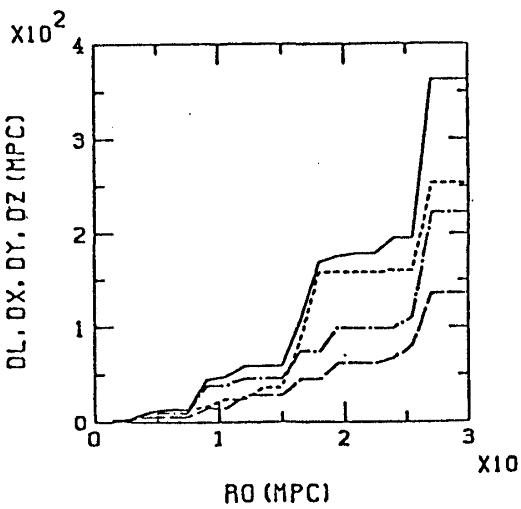

Fig.2

IV. CONCLUSIONS

In Table 1, the effectiveness of each analysis introduced here is shown. The two-point correlation function is a powerful method which gives fairly precise information on the geometry. The nearest neighbor distribution is a good measure for higher order clustering, and the pair separation distribution is good at extracting systematic elongation or velocity dispersion. A new application of the percolation theory we proposed here is an effective method for exploring the directionality as well as the geometry of structure. If these analyses are combined with each other, we can analyze the structures more precisely and quantitatively.

Table 1: Effectiveness of Each Analysis

	$\xi(r)$	$g(r)$	Pair	Percolation
Geometry	good	good	possible	possible
Elongation	—	—	good	good
Directionality	—	—	possible	good

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